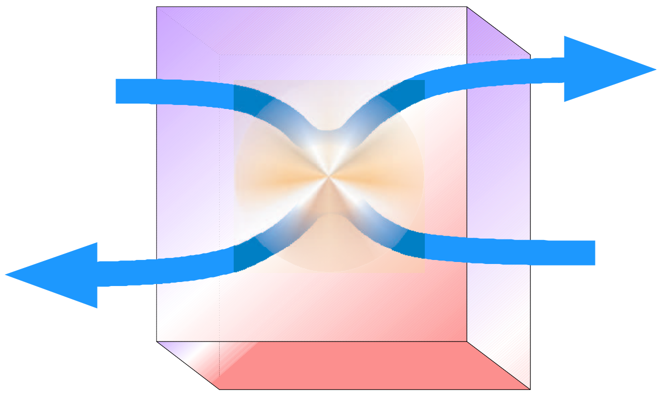
ANOMALOUS COUPLING IN COUNTERSTREAMING PLASMAS



An overview of the physics of “turbulent coupling” between counterstraming collisionless plasmas. Anomalous transport coefficients for various relevant instability mechanisms are given in a form suitable for use in a multifluid code. Cases with and without a magnetic field are considered.

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INSTABILITIES IN COUNTERSTREAMING PLASMAS

Here we reproduce the threshold conditions for the set of instabilities that effect the coupling of counterstreaming plasmas. Since the Lampe, Manheimer, and Papadopoulos (1975) report is no longer readily available, this summary is reasonably complete. These "threshold" conditions as well as the anomalous transport coefficients were derived utilizing kinetic theory but were formulated in terms of fluid quantities in order to permit their use in multifluid and hybrid numerical codes. See also, e.g., Hasegawa (1975) for a description of the Buneman instability, and Stringer (1964) for a description of the ion-acoustic mechanism. The quantities which will be used in the discussion of the instability mechanisms are defined in Table I.

U_{ij}	$\lvert U_i - U_j \rvert$	counterstreaming ion velocity
U_{ie}	$\lvert U_i - U_e \rvert$	U_e includes currents $\perp U_i$
$\bar{v}_i$	$(T_i/m_i)^{1/2}$	ion thermal speed, species i
$\bar{v}_e$	$(T_e/m_e)^{1/2}$	electron thermal speed
ω_{pi}	$(4\pi n_i Z_i^2 e^2/m_i)^{1/2}$	ion plasma frequency, species i
ω_{pj}	$(4\pi n_j Z_j^2 e^2/m_j)^{1/2}$	ion plasma frequency, species j
ω_{pe}	$(4\pi n_e e^2/m_e)^{1/2}$	electron plasma frequency
c_{si}	$\omega_{pi}/\omega_{pe}\ \bar{v}_e$	ionacoustic speed, species i
V_{Ai}	$B/(4\pi n_i m_i)^{1/2}$	Alfven speed, species i
β_e	$8\pi n_e T_e/B^2$	electron beta
Ω_{ci}	$Z_i eB/m_i c$	ion gyrofrequency, species i
Ω_{ce}	$eB/m_e c$	electron gyrofrequency
α_{ji}	$\omega_{pj}^2/\omega_{pi}^2$	by definition, $\alpha_{ji} \leq 1$

Table I. Quantities used in the description of threshold conditions and effective collision frequencies for the instability mechanisms.

Lampe, Manheimer, and Papadopoulos (1975) employ the notation convention that when the (ij) pair of ion streams interacts, $\omega_{pi} \geq \omega_{pj}$.

MOMENTUM COUPLING INSTABILITIES: THE SIMPLE STORY

Only one instability mechanism can cause strong momentum coupling between counterstreaming ion beams: the *ion-ion instability*. In order for this instability to occur, the electrons must be sufficiently hot and the ions sufficiently cool. Avoidance of stabilization by electron shielding requires that

$$U_{ij} \; < \; 1.5\frac{\bar{v}_e \; \omega_{pi}}{\omega_{pe}}(1 \; + \; \alpha_{ji}^{1/3})^{3/2} \sim \; 1.5c_{si}. \qquad \text{Want Hot Electrons}$$

The meaning of this condition can be seen from Figure 1. If the electron velocity sufficiently overlaps the ion distributions, the shielding is supressed. Thus, in the center-of-mass frame, the local Mach number of the streams must be sufficiently small.

For a magnetized plasma, this stabilization condition can be generalized to

$$\begin{aligned} U_{ij}^2 \; &< \; 1.5(n_i/n_e)^2 \; Z_i(1 \; + \; \alpha_{ji}^{1/3})^3 \; V_{Ai}^2(1 \; + \; \beta_e) \\ &\sim \; 1.5(V_{Ai}^2 + c_{si}^2)/Z_i, \qquad \text{Or Large B} \end{aligned}$$

which means that the local *magnetosonic* Mach number must be sufficiently small.

When the ion-ion mechanism is operative, the kinetic energy of the counterstreaming ions gets converted into ion thermal energy. Then the ion velocity space might look like Figure 2. Note that the mean velocities of the two streams have been brought closer together. When the ion velocity distributions overlap sufficiently, the mechanism shuts off. The turn-on condition takes the form

$$U_{ij} \; \geq \; 2\bar{v}_i. \qquad \text{and Cool Ions}$$

This is the story in a nutshell. However the full story is much more complicated. The ion-ion instability must coexist with other instability mechanisms (some of which are closely related to the ion-ion mechanism). These instabilities can help or hinder the ion-ion coupling. In addition, there are complicating factors which arise from the formation of a coupling shell (which will be described below).

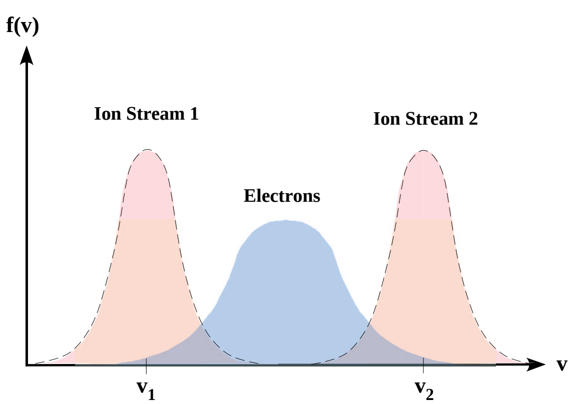

Figure 1. Velocity space configuration of two counterstreaming ion beams with an overlapping electron distribution.

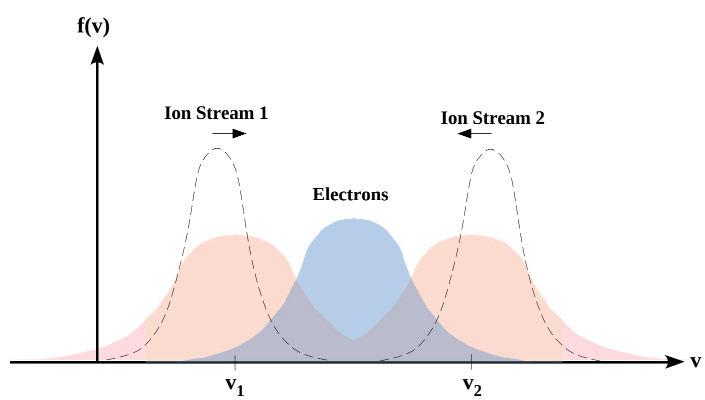

Figure 2. Velocity space configuration of the ion streams after some coupling has occurred. The ions are now hotter, and their distributions have begun to overlap.

NUMERICAL SIMULATION OF COUPLING INSTABILITIES

The physics of "turbulent coupling" (sometimes called "anomalous coupling" or "collisionless coupling") has been discussed in some detail in the plasma physics literature (Stringer 1964; Papadopoulos *et al.* 1971; Landau 1972; Ott *et al.* 1972; McBride *et al.* 1972; Clark, Denevit and Papadopoulos 1973; Book and Clark 1973). The coupling between the counterstreaming ion beams cannot be treated with an ordinary single-fluid model. The momentum transfer and heating due to the various instability processes can be obtained from detailed numerical simulations involving multifluid (Liewer and Krall 1973; Liewer and Davidson 1977; Spicer and Clark 1984) or hybrid models (Clark, Denavit and Papadopoulos 1973; Chodura 1975; Liewer 1976; Sgro and Nielson 1976; Leroy *et al.* 1982). These instabilities lead to collective fluctuating coherent electric fields which can scatter the various plasma components via anomalous collisions. Momentum exchange takes the form

$$\frac{\partial \vec{U}_i}{\partial t} \sim \nu_{ij}(\vec{U}_j - \vec{U}_i),$$

and heating of the ion species takes the form

$$\frac{\partial T_i}{\partial t} \sim \nu_{ij}(\vec{U}_j - \vec{U}_i) \cdot (\vec{V}_{ph} - \vec{U}_i),$$

where ν_{ij} is the effective collision frequency for the instability and $\vec{V}_{ph}$ is the phase velocity of the unstable plasma wave.

ION-ION COUPLING IN AN UNMAGNETIZED PLASMA

There are basically three mechanisms that must be considered in the absence of a magnetic field. The first is the classical unmagnetized ion-ion two-stream instability (UII). Stringer (1964) described this strong momentum-coupling mechanism, and a introductory discussion of the physics involved can be found in Chen (1974). The UII requires that $U_{ij} \leq c_s$ in order to be excited. Thus, the electrons need to be sufficiently hot. The other two instabilities are the Buneman instability (BI) and the ion-acoustic instability (IA) (see, e.g., Hasegawa 1975). These mechanisms are closely related, and can evolve into one another. They are important chiefly for their role in heating electrons (and ions).

I. BUNEMAN THRESHOLD CONDITIONS

For the Buneman instability to be excited, the ratio of the electron to the ion temperature

$$T_e/T_i \simeq 1.0, \tag{BI.1}$$

and the relative drift speed between electrons and ions must satisfy

$$U_{ie} \geq \bar{v}_e. \tag{BI.2}$$

If these conditions are satisfied, the electrons will be heated rapidly to $T_e \approx m_e U_{ie}^2/2$, and the ions will be heated to a lesser extent, until the Buneman instability saturates by electron trapping. The ion-acoustic instability requires $T_e/T_i > 1$ and $U_{ie} \geq c_s\, \Pi_{LD}$, where $\Pi_{LD} = [1 + (m_i/m_e)^{1/2}(T_e/T_i)^{3/2}\; exp(-[T_e/2T_i] - 3/2)]$ results from Landau damping of the ion-acoustic waves. Thus, the BI can naturally provide the necessary conditions for the IA to occur. In turn, these electron heating mechanisms may enable the momentum coupling UII instability to get started.

II. ION-ACOUSTIC THRESHOLD CONDITIONS

Threshold conditions for the ion-acoustic instability, arising from the counterstreaming of electrons with the ith ion stream, are

$$\frac{T_e}{T_i} > 3\, \frac{n_e}{n_i Z_i^2} \qquad (IA.1)$$

and, to avoid Landau damping of the ion-acoustic waves,

$$\frac{U_{ie}}{\bar{v}_e} > \frac{\omega_{pi}}{\omega_{pe}} + \left[\frac{\omega_{pi}\bar{v}_e}{\omega_{pe}\bar{v}_i}\right]^3 exp\Big(-\frac{1}{2}\left[\frac{\omega_{pi}\bar{v}_e}{\omega_{pe}\bar{v}_i}\right]^2 - \frac{3}{2}\Big) \qquad (IA.2)$$

A third requirement for the ion-acoustic instability is that the ion stream i be not UII unstable with respect to any other ion stream j:

$$U_{ij} > 2\left[\frac{\omega_{pj} + \omega_{pi}}{\omega_{pe}}\right] \bar{v}_e. \qquad (IA.3)$$

III. UNMAGNETIZED ION-ION THRESHOLD CONDITIONS

The Unmagnetized Ion-Ion fluid instability is analogous to the classical ion-ion instability, but with generalized threshold conditions (Stringer 1964). The hydrodynamic instability conditions are

$$U_{ij} \geq 2\bar{v}_i \qquad (UII.1)$$

and

$$U_{ij} \geq 2\bar{v}_j\, \alpha_{ji}^{-1/3}. \qquad (UII.2)$$

Avoidance of stabilization by electron shielding requires

$$U_{ij} < 1.5\frac{\bar{v}_e\, \omega_{pi}}{\omega_{pe}}(1 + \alpha_{ji}^{1/3})^{3/2}. \qquad (UII.3)$$

Condition (UII.1) is similar to the condition $U_{ij} = 2.6\ (T_i/m_i)^{1/2}$ found for the classical ion-ion two-stream instability for equal beams (Stringer 1964), and condition (UII.2) allows for the case $T_j \neq T_i$ and $\omega_{pj} < \omega_{pi}$. Condition (UII.3) is similar to the condition $U_{ij} = 2(T_e/m_e)^{1/2}$ and accounts for the role of the electrons in neutralizing the ion space charge created by the ion counterstreaming interaction. The electrons are prevented from neutralizing the space charge only if the preferred wavelength of the ion-ion instability ($\approx U_{ij}/\omega_{pi}$) is less than the electron Debye length $\bar{v}_e/\omega_{pe}$.

ION-ION COUPLING IN A MAGNETIZED PLASMA

Several things change with the addition of a magnetic field. First, several new mechanisms must be considered (they are, naturally, closely related to the instability mechanisms already discussed). Second, an expanding plasma will sweep up the ambient magnetic field, and produce a diamagnetic bubble, as pictured in Figure 3. The magnetic field becomes concentrated in a relatively thin layer, sometimes called the *coupling shell.* The laminar electric field in the coupling shell slows down the expanding plasma and facilitates the onset of the magnetized ion-ion instability (MII). The MII instability requires that $U_{ij} \leq V_{Aj}(1+\beta_j)^{1/2}$, so that either hot electrons or magnetic compression is needed in the region of instability. As in the case with no magnetic field, the ion-ion instabilities provide the strongest momentum coupling between the counterstreaming plasmas. Two additional electron-ion instabilities can occur in a magnetized plasma. They are the beam-cyclotron instability (BCI), and the modified two-stream instability (MTS).

IV. MAGNETIZED ION-ION THRESHOLD CONDITIONS

If conditions (UII.1) and (UII.2) are satisfied, but U_{ij} is greater than condition (UII.3), the magnetized ion-ion instability turns on if

$$U_{ij}^2 \ < \ 1.5(n_i/n_e)^2 \ Z_i(1 \ + \ \alpha_{ji}^{1/3})^3 \ V_{Ai}^2(1 \ + \ \beta_e). \qquad (MII.3)$$

This condition is a requirement to avoid electromagnetic stabilization of the plasma waves. A further turn-on condition for the magnetized ion-ion instability is that the system size, L, be large compared with the wavelength (parallel to B) of the instability. This requires

$$L \ > \ \frac{4.44\ U_{ij}}{(\Omega_{ci}\Omega_{cj})^{1/2}} \ \frac{n_e}{(n_i n_j Z_i Z_j)^{1/2}} \ \frac{1}{(1 \ + \ \alpha_{ji}^{1/3})^{3/2}}. \qquad (MII.4)$$

This condition is usually not very restrictive, as $L \sim R$, the radius of the diamagnetic bubble.

V. MODIFIED TWO-STREAM THRESHOLD CONDITIONS

The modified two-stream instability was originally thought to be an important mechanism for ion momentum coupling. It now appears that it is instead an important heating mechanism in a magnetized plasma. The turn-on requirements for the modified two-stream instability are rather complicated:

$$U_{ie} \ > \ 2\bar{v}_i. \qquad (MTS.1)$$

The hydrodynamic condition also defines the angle of propagation (normal to B) as

$$\theta\frac{\omega_{pe}}{\omega_{pi}} \ \equiv \ \Theta \ = \ min(\Theta_e,\ \Theta_i),$$

where

$$\Theta_e \ = \ \begin{cases} (U_{ie}/4c_{si})^3 & U_{ie} \ \geq \ 4c_{si}, \\ (U_{ie}/2c_{si}) \ - \ 1 & U_{ie} \ < \ 4c_{si}, \end{cases}$$

and

$$\Theta_i \ = \ \begin{cases} (2[1 \ - \ 2\bar{v}_i/U_{ie}])^{3/2} & 2\bar{v}_i \le U_{ie} < 4\bar{v}_i, \\ [U_{ie}/4\bar{v}_i]^{3/2} & 4\bar{v}_i \le U_{ie}, \end{cases}$$

where $c_i \equiv \bar{v}_e \ \omega_{pi}/\omega_{pe}$. To avoid electromagnetic stabilization,

$$\frac{U_{ie}}{V_{Ai}} \ < \ \frac{n_i Z_i}{N_e} \ (1 \ + \ \beta_e)^{1/2} \Big[1 \ + \ \frac{\Theta}{1 \ + \ (1 \ + \ \Omega_{ce}^2/\omega_{pe}^2)^{1/2} \ (1 \ + \ \beta_e)^{1/2}}\Big]. \quad (MTS.2)$$

The requirement that the magnetic field-aligned wavelength be smaller than the system size L is

$$L \ > \ \frac{2\pi U_{ie}}{\omega_{pi}} \Big(1 \ + \ \frac{\omega_{pe}^2}{\Omega_{ce}^2}\Big)^{1/2} \frac{\omega_{pe}}{\omega_{pi}} \frac{1}{\Theta(1+\Theta)}. \quad (MTS.3)$$

The requirement that instability be of an electron-ion type (modified two-stream), rather than ion-ion is

$$\Theta \ > \ \begin{cases} \frac{1}{2}\omega_{pi}/\omega_{pj} & \text{for strong beam i,} \\ \omega_{pi}/\omega_{pj} \quad \omega_{pi}/(\omega_{pi}+\omega_{pj}) & \text{for weak beam j,} \end{cases} \quad (MTS.4)$$

and

$$U_{ie} \ > \ c_{si}. \quad (MTS.5)$$

VI. BEAM-CYCLOTRON THRESHOLD CONDITIONS

The beam-cyclotron instability may be excited if

$$\bar{v}_i \ < \ 2U_{ie} \quad (BC.1)$$

and

$$\bar{v}_e \frac{\omega_{pi}}{\omega_{pe}} \ < \ U_{ie} \quad (BC.2)$$

Since the beam-cyclotron instability undergoes a nonlinear transition to the ion acoustic, it is only important if the ion acoustic is stable, which requires *either*

$$\frac{T_e}{T_i} \ < \ 3\frac{n_e}{n_i Z_i^2}. \quad (BC.3)$$

or

$$\frac{U_{ie}}{\bar{v}_e} \ < \ \frac{\omega_{pi}}{\omega_{pe}} + \Big[\frac{\omega_{pi}\bar{v}_e}{\omega_{pe}\bar{v}_i}\Big]^3 \ exp\Big(-\frac{1}{2}\Big[\frac{\omega_{pi}\bar{v}_e}{\omega_{pe}\bar{v}_i}\Big]^2 \ - \frac{3}{2}\Big). \quad (BC.4)$$

EFFECTIVE COLLISION FREQUENCIES OF THE INSTABILITIES

Below are listed the collision frequencies as found in Lampe, Manheimer, and Papadopoulos (1975) for the instabilities whose threshold conditions were summarized above, and would be used in a multifluid treatment of the coupling. Note that these formulas apply to situations *well above threshold*; the effective collision frequency will be reduced as threshold is approached. In addition, because the various instability mechanisms enable and disable each other in a continuous fashion, time-averaged electron and ion heating rates and momentum coupling rates can often be estimated from *marginal stability* considerations.

As stated above, momentum exchange between two ion streams can be described in terms of the effective collision frequency ν_{ij}:

$$\frac{\partial \vec{U}_i}{\partial t} \sim \nu_{ij}(\vec{U}_j - \vec{U}_i).$$

Ion-Acoustic Instability:

$$\nu_{ei} = \nu_{ie}\frac{\rho_i}{\rho_e} = \frac{\pi^{1/2}\omega_{pe}}{32T_e^2}\left(m_i\left[\frac{\omega_{pi}}{\omega_{pe}}\bar{v}_e - \left(\frac{3T_i}{m_i}\right)^{1/2}\right]^2\right)^2. \qquad (IA)$$

Unmagnetized Ion-Ion Instability:

$$\nu_{ij} = \nu_{ji}\frac{\rho_j}{\rho_i} = 0.15\omega_{pi}\,\frac{\rho_i\rho_j}{(\rho_i+\rho_j)\rho_t}\,[\alpha_{ji}^{2/3} + 1.37(\alpha_{ji}^{1/3} - \alpha_{ji}^{2/3})]. \qquad (UII)$$

Magnetized Ion-Ion Instability:

$$\nu_{ij} = \nu_{ji}\frac{\rho_j}{\rho_i} = \frac{0.15\omega_{pi}}{(1+\omega_{pe}^2/\Omega_{ce}^2)^{1/2}}\frac{\rho_i\rho_j}{(\rho_i+\rho_j)\rho_t}\,[\alpha_{ji}^{2/3} + 1.37(\alpha_{ji}^{1/3} - \alpha_{ji}^{2/3})]. \qquad (MII)$$

Modified Two-Stream Instability:

$$\nu_{ei} = \nu_{ie}\frac{\rho_i}{\rho_e} \approx 0.2\frac{\rho_i}{\rho_e}\frac{\sqrt{3}/2^{4/3}\omega_{pi}}{(1+\omega_{pe}^2/\Omega_{ce}^2)^{1/2}}\left\{\begin{matrix}\Theta^{1/3} & for\ \Theta > 1\\ \Theta^{2/3} & for\ \Theta < 1\end{matrix}\right\}. \qquad (MTS)$$

Beam-Cyclotron Instability:

$$\nu_{ei} = \nu_{ie}\frac{\rho_i}{\rho_e} = 2.51\frac{\omega_{pe}^2}{\Omega_{ce}}\frac{[U_{ie} - \bar{v}_i]^4}{c^2\bar{v}_e^2}\frac{B^2}{16n_eT_e}\left(1.52\frac{n_iZ_i^2T_e}{n_eT_i}\right)^2. \qquad (BC)$$

PHASE VELOCITIES OF THE INSTABILITIES

Below are listed the phase velocities as found in Lampe, Manheimer, and Papadopoulos (1975) for the instabilities whose threshold conditions were summarized above, and would be used in a multifluid treatment of the coupling. They are used to calculate the ion heating rates for each active mechanism.

Heating of an ion species from the coupling with another ion stream can be expressed in terms of the effective collision frequency ν_{ij} and the phase velocity of the unstable wave V_{ph}:

$$\frac{\partial T_i}{\partial t} \sim \nu_{ij}(\vec{U}_j - \vec{U}_i) \cdot (\vec{V}_{ph} - \vec{U}_i).$$

Ion-Acoustic Instability:

$$\vec{V}_{ph} = \vec{U}_i - c_{si}\,\frac{\vec{U}_{ie}}{U_{ie}}. \qquad (IA)$$

Unmagnetized Ion-Ion Instability:

$$\vec{V}_{ph} = \vec{U}_j - \frac{1}{2}\alpha_{ji}^{1/3}\,\vec{U}_{ie}. \qquad (UII)$$

Magnetized Ion-Ion Instability:

$$\vec{V}_{ph} = \vec{U}_j - \frac{1}{2}\alpha_{ji}^{1/3}\,\vec{U}_{ie}. \qquad (MII)$$

Modified Two-Stream Instability:

$$\vec{V}_{ph} = \left\{ \begin{array}{ll} \vec{U}_e + \frac{1}{2}\Theta^{2/3}\,\vec{U}_{ie} & for\ \Theta < 1 \\ \vec{U}_e + (1 - \frac{1}{2}\Theta^{-2/3})\,\vec{U}_{ie} & for\ \Theta > 1 \end{array} \right\}. \qquad (MTS)$$

Beam-Cyclotron Instability:

$$V_{ph} = U_i - \bar{v}_i. \qquad (BC)$$

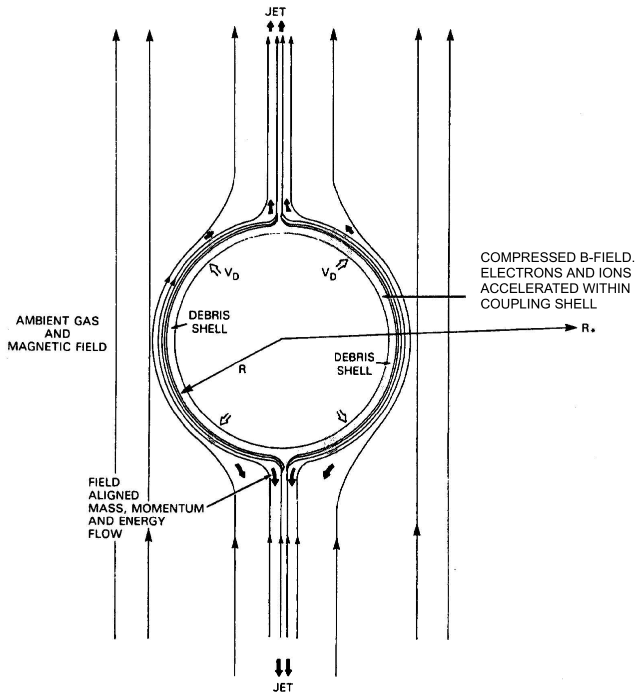

Figure 3. Geometry of the coupling to a background plasma in the presence of an initially uniform magnetic field. The debris plasma initially expands with spherical symmetry. A shell of compressed magnetic field (the "coupling shell") forms. This is the region where most of the electron and ion heating and momentum coupling occur.

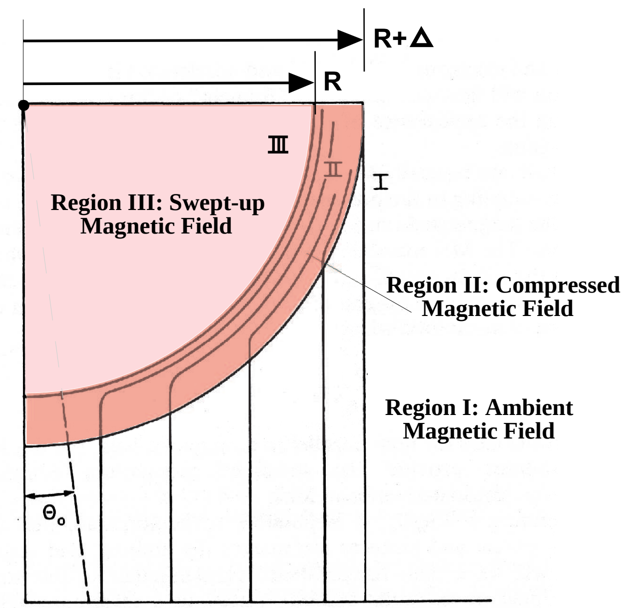

Figure 4. Detail of the Coupling Shell for a magnetized plasma. where Δ is the thickness of the shell. Charged particle loss can occur because coupling is weak along the magnetic field lines (Papadopoulos et al. 1971). The angle θ_0 below which coupling fails can be estimated from $\theta_0 \approx (\Delta/R)^{1/2}$.

COUPLING GEOMETRY IN A MAGNETIZED MEDIUM

In the presence of a magnetic field, strong coupling of the piston ions and the ambient ions can be spatially split up into essentially three distinct regions (Fig. 3 amd 4). Region I contains essentially uncompressed magnetic field, ambient ions, and a small fraction of piston plasma, part of which has moved ahead of the coupling shell (region II) before the turbulent coupling mechanisms commence, and part of which will be accelerated continuously from the coupling region during the expansion. Region II is the coupling region. It contains a highly compressed magnetic field, short-wavelength electrostatic turbulent fields, and a mixture of coupled hot ambient ions and plasma piston. Region III contains essentially no magnetic field because it has been swept out into region II. However, it contains the part of the piston that has not yet interacted, which at early times will have rather high densities.

Coupling is strong in regions where the magnetic field is perpendicular to the expansion and fails in a cone of angles almost parallel to the magnetic field $\theta < \theta_0$, where θ_0 is estimated from $\theta_0 \approx (\Delta/R)^{1/2}$, where Δ is the thickness of the shell.

The unmagnetized ion-ion two-stream instability requires extremely hot electrons (a low Mach number) to be excited. This condition is relaxed for the magnetized ion-ion instability, which requires a low magnetoacoustic Mach number. In addition, the laminar deceleration of the streaming ions in the coupling shell reduces the local Mach number. Finally, there is the phenomenon of Larmor coupling in a magnetized plasma, where the counterstreaming ions exchange momentum through their Larmor motion. Larmor coupling is not a true coupling mechanism, however, because the kinetic energy is not thermalized.

THE CONCEPT OF MARGINAL STABILITY

The plasma instability mechanisms discussed so far typically interact in a complex manner through their respective turnon conditions. As an example, consider the interaction of the ion-acoustic instability and the unmagnetized ion-ion instability in the absence of other mechanisms. When the ion-ion instability is operative, it heats the ions only. However, the ion-acoustic instability heats both ions and electrons according to the prescription (e.g., see Galeev and Sagdeev 1984)

$$\frac{\dot{T}_i}{\dot{T}_e} = \frac{\omega_{pi}\bar{v}_e}{\omega_{pe}U_{ie}} \equiv Z \frac{c_{si}}{U_{ie}}. \qquad (IA.H)$$

Furthermore, the threshold conditions for the ion-ion instability which are relevant this discussion are

$$U_{ij} \geq 2\bar{v}_i, \qquad (UII.1)$$

$$U_{ij} \geq 2\bar{v}_j \, \alpha_{ji}^{-1/3}, \qquad (UII.2)$$

$$U_{ij} < 1.5\bar{v}_e \frac{\omega_{pi}}{\omega_{pe}}(1 + \alpha_{ji}^{1/3})^{3/2}; \qquad (UII.3)$$

while those for the ion-acoustic instability are

$$T_e > 3T_i \frac{n_e}{n_i Z_i^2} \qquad (IA.1)$$

and

$$U_{ie} > c_{si} \left[1 + \frac{c_{si}^2 \bar{v}_e}{\bar{v}_i^3} \, exp\Big(-\frac{c_{si}^2}{2\bar{v}_i^2} - \frac{3}{2}\Big)\right]. \qquad (IA.2)$$

The ion-acoustic instability tends to shut off when the ions get too hot relative to the electrons, and the ion-ion instability heats only the ions. Thus, when the ion- ion instability is operative for a sufficiently long time, it can effectively shut itself off. This can occur in two ways. Either the effective collision frequency of the ion-ion instability becomes smaller as the ion temperature threshold is reached, or the instability will shut off completely until the ions cool. In either case, the system is said to reside in a state of marginal stability. In the latter case, however, there may be an observable cycling on and off of the instability. Furthermore, the ion heating and the momentum coupling of the ion-ion instability can shut off the ion-acoustic instability. However, the ion-acoustic instability is the mechanism for electron heating which allows the ion-ion threshold condition (UII.1,2) to be satisfied. When the ion-acoustic instability is turned off for a sufficiently long time, electrons cool and the ion-ion instability is effectively shut off. This allows the ions to cool and the threshold condition (IA.1) to be satisfied.

In this way, the two instability mechanisms can control each other. The resulting ion and electron temperatures will be effectively maintained near the threshold for turn-on of the relevant instabilities. In this sense, as above, the plasma will remain in a state of marginal stability. In a system with more than two such mechanisms, the situation may be more complicated, Also, transport of mass, momentum, and energy can result in the incidence of an instability in one place enabling or disabling an instability elsewhere.

Because the coupling instabilities tend to reside near marginal stability, and are constantly being turned on (and turned off), the relevant parameters are often not the respective *collision frequencies*. Rather, they are likely to be those which describe the *turn-on conditions* for the coupling mechanisms. It will often be found in practice that ion and electron temperatures are fixed at values near an operative threshold condition.

IONIZATION OF THE PLASMA STREAMS

The mechanisms described in this summary require ionized plasmas. If the background material is a neutral gas, coupling will not occur. However, thre are processes which can ionize the ambient material. One such process is radiation. If UV or x-ray radiation is produced in the experiment, it will cause some preionization of the gas. If the gas density is high enough, the radiation will be absorbed, and there will be some radius R_{ion} beyond which the gas is essentially unionized. Another mechanism is ionization due to the interaction with the hot coupling shell. If coupling commences due to radiative preionization, it can be continued through collisional ionization in the coupling region. The efficiency of these processes can be estimated for the parameters of the experiment. However, when these ionization mechanisms are ineffective, the anomalous coupling becomes ineffective.

Furthermore, the strength of the coupling ν_{ij} is a function of the ionization of the plasma streams, Z_i and Z_j. If there are multiple ionized species for either or both of the streams, separate ion fluids may be required for each species. As the plasmas ionize, material is transferred to fluids representing higher charge states. In addition, each fluid will be affected differently by the local electric field $\vec{E}$ and magnetic field $\vec{B}$.

PLASMA FLOW NEAR THE COUPLING REGION

The plasma flow in the vicinity of the coupling region is complex, but some general observations may be made. The governing equations in a multifluid description are

Ion continuity equation:

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot (n_i \vec{v}_i) \qquad (F.1)$$

Ion momentum equation (where $\rho_i = n_i m_i$):

$$\frac{\partial(\rho_i \vec{v}_i)}{\partial t} = -\nabla \cdot (\rho_i \vec{v}_i \vec{v}_i) - \nabla P_i + Z_i e n_i \big(\vec{E} + \frac{\vec{v}_i \times \vec{B}}{c}\big) + \sum_j \rho_j \nu_{ij} (\vec{v}_j - \vec{v}_i) \qquad (F.2)$$

Electron momentum equation (where $\rho_e = n_e m_e$):

$$\frac{\partial(\rho_e \vec{v}_e)}{\partial t} = -\nabla \cdot (\rho_e \vec{v}_e \vec{v}_e) - \nabla P_e - e n_e \big(\vec{E} + \frac{\vec{v}_e \times \vec{B}}{c}\big) + \sum_j \rho_e \nu_{ej} (\vec{v}_j - \vec{v}_e) \qquad (F.3)$$

Ion and Electron temperature equations (neglecting thermal conduction and radiation):

$$\frac{\partial T_s}{\partial t} = -\nabla \cdot (T_s \vec{v}_s) - (\gamma - 2) T_s \nabla \cdot v_s - (\gamma - 1) \sum_j m_s \nu_{sj} (\vec{v}_{ph} - \vec{v}_s) \cdot (\vec{v}_j - \vec{v}_s) \qquad (F.4)$$

where the reference ion i counterstreams through multiple ion streams j. These fluid equations can be combined with Maxwell's equations

Ampere's Law (neglecting $\partial \vec{E}/\partial t$):

$$\frac{c}{4\pi} \nabla \times \vec{B} = \vec{J} = -n_e e \vec{v}_e + \sum_i Z_i e n_i \vec{v}_i \qquad (M.1)$$

Faraday's Law:

$$\frac{\partial \vec{B}}{\partial t} = -c \nabla \times \vec{E} \qquad (M.2)$$

The coupling terms involving ν_{ij} refer to a single instability mechanism, but a simple summation over all relevant processes can be performed to generalize these expressions. γ is the gas constant, and the pressure $P_s = n_s T_s / (\gamma - 1)$.

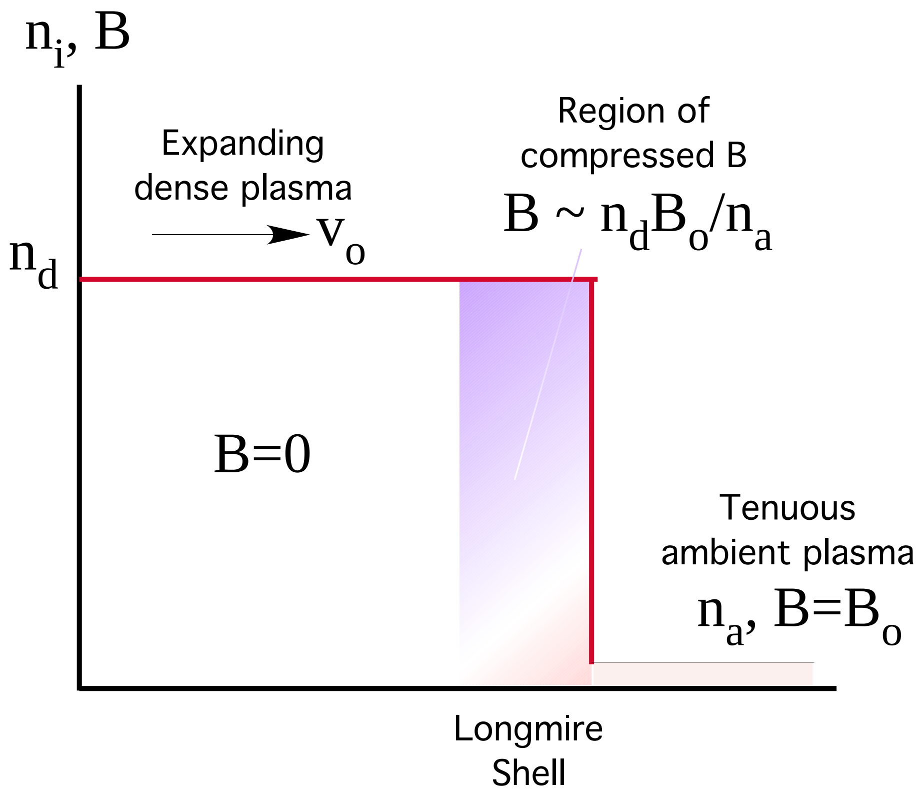

Figure 5. Schematic representation of the Longmire shell. Ambient magnetic field B_o, here assumed perpendicular to the debris expansion, is swept up by the debris ions and compressed in a narrow layer. The compressed field can be as large as $B \sim B_0(n_d/n_a)$, but resistivity and plasma acceleration near the shell will limit the compression.

In situations where the debris ions expand across a magnetic field, a shell of compressed magnetic field, sometimes called the Longmire shell, can be formed. This occurs because the magnetic field lines are dragged along by the ions. If we ignore the electron inertia terms, Equation (F.3) reduces an expression for the electric field

$$en_e\vec{E} \;=\; -\,\nabla P_e \;-\; en_e\Big(\frac{\vec{v}_e \times \vec{B}}{c}\Big) \;+\; \sum_j n_e m_e \nu_{ej}(\vec{v}_e - \vec{v}_j). \qquad (F.3R)$$

Then, substituting this expression, Equation (M.2) becomes

$$\frac{\partial \vec{B}}{\partial t} \;=-\frac{c}{en_e}\nabla \times \nabla P_e \;+\; \nabla \times (\vec{v}_e \times \vec{B}) \;+\; \frac{m_e c}{e}\sum_j \nu_{ej}\nabla \times (\vec{v}_e - \vec{v}_j).$$

The pressure gradient term vanishes, and if the resistivity term is neglected, Equation (M.2) simplifies to

$$\frac{\partial \vec{B}}{\partial t} \;=\; +\,\nabla \times (\vec{v}_e \times \vec{B}),$$

which, for flow perpendicular to the magnetic field, becomes

$$\frac{\partial B}{\partial t} \;=\; -\nabla \cdot (B\vec{v}_i)$$

which is identical in form to Equation (F.1). Thus, in this case B/n_i is constant, and the magnetic compression will be approximately equal to the debris/ambient density ratio.

Equations (F.2) and (F.3R) can be combined, solving for $\vec{E}$ to give

$$\frac{\partial(\rho_i \vec{v}_i)}{\partial t} = -\nabla \cdot (\rho_i \vec{v}_i \vec{v}_i) - \nabla(P_i + \frac{Z_i n_i}{n_e} P_e) + \Big(\frac{\vec{J} \times \vec{B}}{c}\Big) + \sum_{j \neq i} Z_j e n_j \Big(\frac{\vec{v}_j \times \vec{B}}{c}\Big) + \sum_j \rho_j \nu_{ij} (\vec{v}_i - \vec{v}_j) - Z_i n_i m_e \sum_j \nu_{ej} (\vec{v}_e - \vec{v}_j).$$

But from (M.1),

$$\frac{1}{c}(\vec{J} \times \vec{B}) = \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} = \frac{1}{4\pi}\vec{B} \cdot \nabla \vec{B} - \frac{1}{8\pi}\nabla \vec{B}^2.$$

For the situation where the debris density predominates ($n_i >> n_j$), the momentum equation can be simplified:

$$\frac{\partial(\rho_i \vec{v}_i)}{\partial t} = -\nabla \cdot (\rho_i \vec{v}_i \vec{v}_i) - \nabla(P_i + P_e + \frac{1}{8\pi}\vec{B}^2) + \frac{1}{4\pi}\vec{B} \cdot \nabla \vec{B} + \sum_j \rho_j \nu_{ij} (\vec{v}_i - \vec{v}_j) - Z_i n_i m_e \sum_j \nu_{ej} (\vec{v}_e - \vec{v}_j).$$

The compressed magnetic field contributes to the pressure gradient term, and causes laminar deceleration on one side of the coupling shell (or Longmire shell) and acceleration on the other side. In the absence of a magnetic field, the thermal pressure gradients produce a similar (but less pronounced) effect.

What does the plasma look like in the vicinity of the coupling shell? For a two component (debris/ambient) plasma, the [V,R] phase space resembles the schematic plot in Figure 6. This phase space configuration is due to laminar acceleration (and deceleration) of the plasmas by electric fields in the vicinity of the coupling shell, and is quite complex. Four main plasma streams form outside of the coupling region, labeled I-IV. The debris plasma (I) with velocity V_o streams through the ambient plasma (II). Some fraction of the debris plasma escapes coupling and is laminarly accelerated ahead of the coupling shell. This is designated the *precursor plasma* (III) in the plot. It can have a velocity exceeding twice the debris expansion velocity. As the plot indicates, it is a hot plasma, with an ion thermal energy comparable to the (unaccelerated) ion kinetic energy. The corresponding uncoupled ambient plasma (IV) is also hot, and can be accelerated radially inwards. It can be thought of as an ambient precursor plasma.

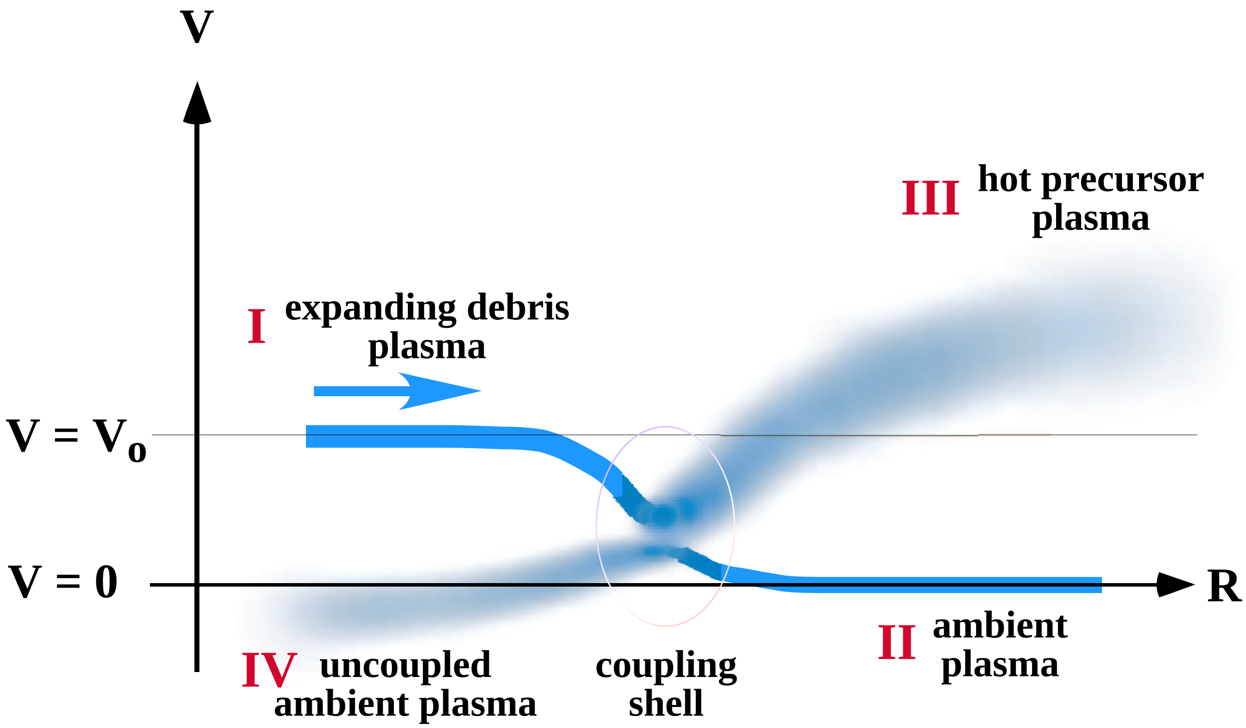

Figure 6. Schematic representation of the debris-ambient phase space in the vicinity of the coupling shell, illustrating the laminar acceleration of the plasma streams by the large electric fields in the coupling region. The radial electric field is produced by thermal (and magnetic field) gradients in the coupling shell.

An additional complication in a magnetic field is the $\vec{v} \times \vec{B}$ motion of the ions in and ahead of the coupling shell. The hot precursor ions will momentum couple with the ambient plasma through the Larmor motion of the counterstreaming ions. This phenomenon is called *Larmor Coupling*, and it must be properly modeled, but it will not thermalize the kinetic energy of the precursor ions. It is therefore problematic in a multifluid code to treat this phenomenon. The modeling of the plasma components shown in Figure 6 is likewise difficult.

A FINAL CAVEAT

Many of the approximations and schematic pictures discussed above depend on the plasma parameters (plasma densities, magnetic field, flow velocities, geometry, timescales, etc.), and substantially different phenomena can result in different circumstances. Thus, this brief summary of coupling physics, which was developed for high-altitude nuclear environments, and which was subsequently applied to supernova phenomena, may not apply in other situations. The numbers have to be *crunched*.

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